Total marks – 70

QUESTION 1-10 Objective-response questions. *(10marks)*

Write answers on the Objective response answers booklet **Marks**

Multiple Choice

1. Which one of the following is NOT a polynomial: **1**

(A) $\left(\sqrt{3}x - 1\right)^3$. (B) $\dfrac{x + 3}{5}$

(C) $\dfrac{1}{x^2} + 4x + 7$ (D) $-2x^3 + 0.5x^2$

2. The value of k if $x + 1$ is a factor of $x^3 - 2x^2 - kx + 5$ is: **1**

(A) 4 (B) 2

(C) -2 (D) -6

3. How many solutions would $\cos^2\theta = \dfrac{1}{4}$ have in the domain $0 \leq \theta \leq 2\pi$? **1**

(A) 0. (B) 2 (C) 1 (D) 4

4. Which of the following gives the correct simplification of $\sin x \sec x$. **1**

(A) 1. (B) $\cot x$ (C) $\tan x$ (D) $\cos ecx$

5. Find the gradient of the normal to the parabola $x^2 = 4ay$ at $\left(2at, at^2\right)$. **1**

(A) t (B) $-\dfrac{1}{t}$ (C) $\dfrac{1}{t}$ (D) $-t$

6. The exact value of $\tan^{-1}\left(\tan\dfrac{2\pi}{3}\right)$ is **1**

(A) $-\dfrac{\pi}{3}$ (B) $-\sqrt{3}$ (C) $-\dfrac{2\pi}{3}$ (D) $\dfrac{2\pi}{3}$

7. Which of the following is the correct expansion for $\cos\left(\theta - \dfrac{\pi}{3}\right)$ **1**

(A) $\dfrac{1}{2}\left(\cos\theta - \sqrt{3}\sin\theta\right)$ (B) $\cos\theta - \dfrac{1}{2}$

(C) $\dfrac{1}{2}\cos\theta$ (D) $\dfrac{1}{2}\left(\cos\theta + \sqrt{3}\sin\theta\right)$

8. Which of the following is the correct general solution for $\sin x = \dfrac{1}{2}$. **1**

(A) $x = n\pi + (-1)^n\dfrac{\pi}{6}$ (B) $x = n\pi + \dfrac{\pi}{6}$

(C) $x = 2n\pi + \dfrac{\pi}{6}$ (D) $x = 2n\pi \pm \dfrac{\pi}{6}$

9. Which of the following statements is FALSE. **1**

(A) $\cos^{-1}(-\theta) = -\cos^{-1}\theta$. (B) $\sin^{-1}(-\theta) = -\sin^{-1}\theta$

(C) $\tan^{-1}(-\theta) = -\tan^{-1}\theta$ (D) $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

10. Consider the function $f(x) = x^2 - 6x$. Which of the following gives the correct domain of $f(x)$ for which there exists an inverse function, $f^{-1}(x)$. **1**

(A) *All real x.* (B) $0 \le x \le 6$

(C) $x \le 3$ *or* $x \ge 3$ (D) $x \ge 0$

End of Question Multiple Choice

QUESTION 11 *(15 marks)* ***START A NEW BOOKLET*** **Mark**

(a) Find the acute angle between the lines: $x - 2y + 1 = 0$ and $y = 5x - 4$, to the nearest degree. **3**

(b) Solve the inequality $\frac{4}{x+1} \leq 3$ and graph your solution on the number line. **3**

(c) Find the coordinates of the point P which divides the interval joining A(-4, 3) and B(-8, -9) externally in the ratio 2 : 3. **2**

(d) Use the method of mathematical induction to prove that

$$1 + 3 + 9 + \ldots\ldots\ldots\ldots\ldots + 3^{n-1} = \frac{1}{2}\left(3^n - 1\right) \text{ for integer } n \geq 1$$

4

(e) Find the cartesian equation of the curve represented by the following parametric equations;

$$x = 5\sin\theta$$
$$y = 5\cos\theta \quad .$$

3

End of Question 11

QUESTION 12 *(15 marks)* ***START A NEW BOOKLET*** **Marks**

(a) (i) Write $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0 \le x \le \frac{\pi}{2}$. **1**

(ii) Hence, or otherwise solve the equation $\sin x - \sqrt{3}\cos x = 1,\ \ 0 \le x \le 2\pi$. **2**

(b) Evaluate $\int_0^{\frac{\pi}{4}} \cos^2(2x)\,dx$. **2**

(c) Using the substitution $t = \tan\frac{\theta}{2}$,

(i) Write down an expression for $\tan\theta$, in terms of t. **1**

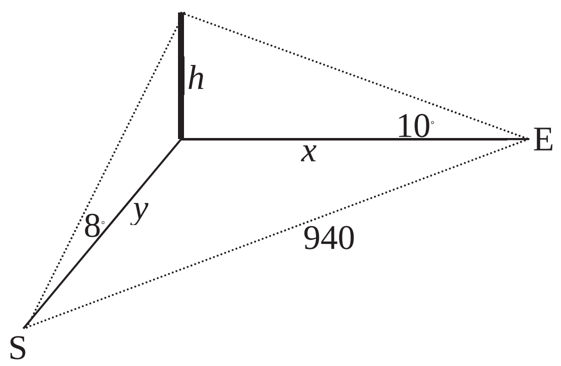
(ii) Use this expression to show $\tan 15^\circ = 2 - \sqrt{3}$ **2**

(d) Prove $\frac{\sin 3A}{\sin A} + \frac{\cos 3A}{\cos A} = 4\cos 2A$. **3**

(e) A surveyor who is y metres south of a tower sees the top of it with an angle of elevation 8°. A second surveyor is x metres east of the tower. From his position the angle of elevation is 10° to the top of the tower. The two surveyors are 940m apart.

(i) Show that $y = h\cot 8^\circ$ **1**

(ii) Find the height of the tower to the nearest metre. **3**



End of Question 12

QUESTION 13 *(15 marks)* ***START A NEW BOOKLET*** **Marks**

(a) Find the exact values of each of the following, showing all working.

(i) $\cos^{-1}\left(\sin\frac{\pi}{3}\right)$ **1**

(ii) $\sin\left(2\tan^{-1}\frac{1}{3}\right)$. **2**

(b) Find $\frac{d}{dx}(x^2\cos^{-1}2x)$ **2**

(c) Find $\int\frac{-1}{\sqrt{1-4x^2}}\,dx$ **2**

(d) Evaluate $\int_0^1\frac{5}{3+x^2}\,dx$ **2**

(e) (i) Find the domain and range for the function $y = 5\sin^{-1}\left(\frac{x}{\pi}\right)$. **2**

(ii) Sketch $y = 5\sin^{-1}\left(\frac{x}{\pi}\right)$. **2**

(f) For what values of x does $y = \frac{xe^x}{2}$ have an inverse function. Use differentiation to explain your answer. **2**

End of Question 13

QUESTION 14 *(15 marks)* ***START A NEW BOOKLET*** **Marks**

(a) The polynomial $p(x) = x^3 + ax + b$ has $(x-5)$ as one of its factors and has a remainder of -60 when divided by $(x+5)$. Find the values of a and b. **3**

(b) The polynomial equation $x^3 - 5x^2 + 7x + 5 = 0$ has 3 roots, $\alpha,\ \beta,\ \gamma$

(i) Find $\alpha + \beta + \gamma$ **1**

(ii) Find $\alpha\beta + \beta\gamma + \gamma\alpha$ **1**

(iii) Find $\alpha^2 + \beta^2 + \gamma^2$ **2**

(c) Given that $x^3 - 3x^2 + 2x - 4 = \left(x^2 + 2\right) \times Q(x) + R(x)$. **3**

Find $Q(x)$ and $R(x)$.

(d) Sketch the following polynomial. Clearly show all intercepts.

$$P(x) = x(x+2)^2(1-x)^3.$$ **2**

(e) Solve the equation $3x^3 - 17x^2 - 8x + 12 = 0$ given that the product of two of the roots is 4. **3**

END OF THE PAPER

Year 12 Extension 1Mathematics Assessment 2 2012

Student Number: _______ANSWERS___________________________

Questions 1-10 Multiple Choice Answer Sheet

1.	A ○	B ○	C ●	D ○
2.	A ○	B ○	C ●	D ○
3.	A ○	B ○	C ○	D ●
4.	A ○	B ○	C ●	D ○
5.	A ○	B ●	C ○	D ○
6.	A ●	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ●
8.	A ●	B ○	C ○	D ○
9.	A ●	B ○	C ○	D ○
10.	A ○	B ○	C ●	D ○

11(*a*)

$x - 2y + 1 = 0 \quad \therefore m_1 = \frac{1}{2} \quad$ *for* $\quad y = 5x - 4 \quad \therefore m_2 = 5 \quad \surd$

$u \sin g \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1.m_2}\right|$

$$\therefore \tan\theta = \left|\frac{\frac{1}{2} - 5}{1 + \frac{5}{2}}\right| = \left|\frac{-9}{7}\right| \qquad \surd$$

$$\therefore \theta = \quad 52^0 \quad (\textit{nearest}\ \deg)\surd$$

11(*b*)

$\frac{4}{x+1} \le 3$

$\therefore 4(x+1) \le 3(x+1)^2$

$\therefore 3(x+1)^2 - 4(x+1) \ge 0$

$(x+1)[3x + 3 - 4] \ge 0$

$(x+1)(3x-1) \ge 0$

$\therefore x \ge \frac{1}{3} \quad or \quad x < -1 \qquad \surd\surd\surd$

11(c) A(-4, 3) and B(-8, -9)
Externally so use 2 : -3

$x = \frac{-3(-4) + 2(-8)}{2 + (-3)} \qquad y = \frac{-3(3) + 2(-9)}{2 + (-3)}$

$= \frac{12 - 16}{-1} \qquad = \frac{-9 - 18}{-1}$

$= \frac{-4}{-1} \qquad = \frac{-27}{-1}$

$= 4 \qquad = 27 \qquad \surd\surd$

P is the point (4, 27)

11(d)

$$1 + 3 + 9 + \ldots\ldots\ldots\ldots\ldots + 3^{n-1} = \frac{1}{2}\left(3^n - 1\right)$$

Test n = 1. $3^{1-1} = \frac{1}{2}\left(3^1 - 1\right) = 1$ $\therefore$ true for $n = 1$ √

$n = k$ *i.e.* $1 + 3 + 9 + \ldots\ldots\ldots\ldots\ldots + 3^{k-1} = \frac{1}{2}\left(3^k - 1\right)$

Prove true for n=k + 1

i.e. $1 + 3 + 9 + \ldots\ldots\ldots\ldots\ldots + 3^{k-1} + 3^k = \frac{1}{2}\left(3^k - 1\right) + 3^{(k+1)-1}$ √

$$= \frac{1}{2}\left(3^k - 1\right) + 3^k$$

$$= \frac{1}{2}\left(3^k - 1 + 2.3^k\right)$$

$$= \frac{1}{2}\left(3.3^k - 1\right)$$

$$= \frac{1}{2}\left(3^{k+1} - 1\right)$$

Which is in the form $\frac{1}{2}\left(3^n - 1\right)$ where $n = k + 1$ √

$\therefore$ True for $n = k + 1$ when true for $n = k$,
But it is true for $n = 1$ $\therefore$ True for $n = 1 + 1 = 2$
And 2 + 1 = 3 Etc.
Hence by Mathematical Induction √(accept any conclusion)

$$1 + 3 + 9 + \ldots\ldots\ldots\ldots\ldots + 3^{n-1} = \frac{1}{2}\left(3^n - 1\right)$$

11(e)

$x = 5\sin\theta$

$y = 5\cos x$

$\therefore x^2 = 25\sin^2\theta$ √

$y^2 = 25\cos^2\theta$

$\therefore x^2 + y^2 = 25(\sin^2\theta + \cos^2\theta)$ √

$\therefore x^2 + y^2 = 25$ √

QUESTION 12 *(15 marks)* ***START A NEW BOOKLET*** **Marks**

(a) (i) If $sin\,x - \sqrt{3}\ cosx = Rsin(x-\alpha)$ then

$$R = \sqrt{1^2 + (\sqrt{3})^2} \quad \text{and} \quad tan\,\alpha = \sqrt{3}$$

$$\therefore R = 2 \quad \text{and} \quad \alpha = \frac{\pi}{3}$$

So $sin\,x - \sqrt{3}\ cosx = 2\,sin\left(x - \frac{\pi}{3}\right)$

1

(ii) Hence, or otherwise solve the equation $\sin x - \sqrt{3}\cos x = 1,\ \ 0 \le x \le 2\pi$.

$$sin\,x - \sqrt{3}\ cosx = 1$$

$$2\,sin\left(x - \frac{\pi}{3}\right) = 1$$

$$sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$$

$$x - \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{2}, \frac{7\pi}{6}$$

12(*b*) $\int_0^{\frac{\pi}{4}} \cos^2(2x)dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(1 + \cos(4x)\right)\cos^2 dx$ √

$$= \frac{1}{2}\left[\left[x + \frac{1}{4}\sin(4x)\right]\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left(\frac{\pi}{4} + 0 - 0\right)$$

$$= \frac{\pi}{8}$$ √

12(c) (i) $t = \tan\frac{\theta}{2}$

$\therefore \tan\theta = \frac{2t}{1 - t^2}$ √

(ii) $\therefore \tan 30^{\circ} = \frac{2t}{1 - t^2}$ √

$\therefore 1 - t^2 = 2\sqrt{3}t$

$\therefore t^2 + 2\sqrt{3}t - 1 = 0$

$\therefore t = \frac{-2\sqrt{3} + \sqrt{12 + 4}}{2}$ √

$\therefore t = 2 - \sqrt{3}$

,

12(d) $\frac{\sin 3A}{\sin A} + \frac{\cos 3A}{\cos A} = 4\cos 2A$

$LHS = \frac{\sin 3A}{\sin A} + \frac{\cos 3A}{\cos A}$

$= \frac{\sin 3A\cos A + \cos 3A\sin A}{\sin A\cos A}$ √

$= \frac{\sin(3A + A)}{\sin A\cos A}$

$= \frac{\sin 4A}{\frac{1}{2}\sin 2A}$ √

$= \frac{4\sin 2A\cos 2A}{\sin 2A}$ √

$= 4\cos 2A = RHS$

.

12(e) (i) $\dfrac{h}{y} = \tan 8$ so $\dfrac{y}{h} = \cot 8^\circ$ ie $y = h\cot 8^\circ$

(ii) likewise $x = h\cot 10^\circ$

By Pythagoras $x^2 + y^2 = 940^2$ i.e.

$$h^2\cot^2 8^\circ + h^2\cot^2 10^\circ = 883600$$

$$h^2\left(\cot^2 8^\circ + \cot^2 10^\circ\right) = 883600$$

$$h^2 = \frac{883600}{\cot^2 8^\circ + \cot^2 10^\circ} = \frac{883600}{82.7919} = 10672.54$$

$h = 103$ metres (to nearest metre)

End of Question 12

QUESTION 13 *(15 marks)* ***START A NEW BOOKLET*** **Marks**

(a) Find the exact values of

(i) $\cos^{-1}\left(\sin\dfrac{\pi}{3}\right) = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{\pi}{6}$ √

(ii) $\sin\left[2\tan^{-1}\dfrac{1}{3}\right] = 2\sin\theta\cos\theta$ *let* $\theta = \tan^{-1}\dfrac{1}{3}$

$= 2 \times \dfrac{1}{\sqrt{10}} \times \dfrac{3}{\sqrt{10}}$ √

$= \dfrac{3}{5}$ √

(b) Find $\dfrac{d(x^2\cos^{-1}2x)}{dx}$

$\dfrac{d}{dx}(2x^2\cos^{-1}2x) = 2x^2\dfrac{-1}{\sqrt{\frac{1}{4}-x^2}} + 4x\cos^{-1}2x$

$= 2x^2\dfrac{-2}{\sqrt{1-4x^2}} + 4x\cos^{-1}2x$

$= \dfrac{-4x^2}{\sqrt{1-4x^2}} + 4x\cos^{-1}2x$

√ √

13(c) $\displaystyle\int\frac{-1}{\sqrt{1-4x^2}}\,dx = \frac{1}{2}\int\frac{-1}{\sqrt{\frac{1}{4}-x^2}}\,dx$

$= \dfrac{1}{2}\cos^{-1}(2x) + c$ √√ *accept answer only*

13(d) $\int_0^1 \frac{5}{3+x^2}\,dx = \frac{5}{\sqrt{3}}\left[\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right]_0^1$ √

$= \frac{5}{\sqrt{3}}\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - 0\right]$

$= \frac{5}{\sqrt{3}} \times \frac{\pi}{6}$

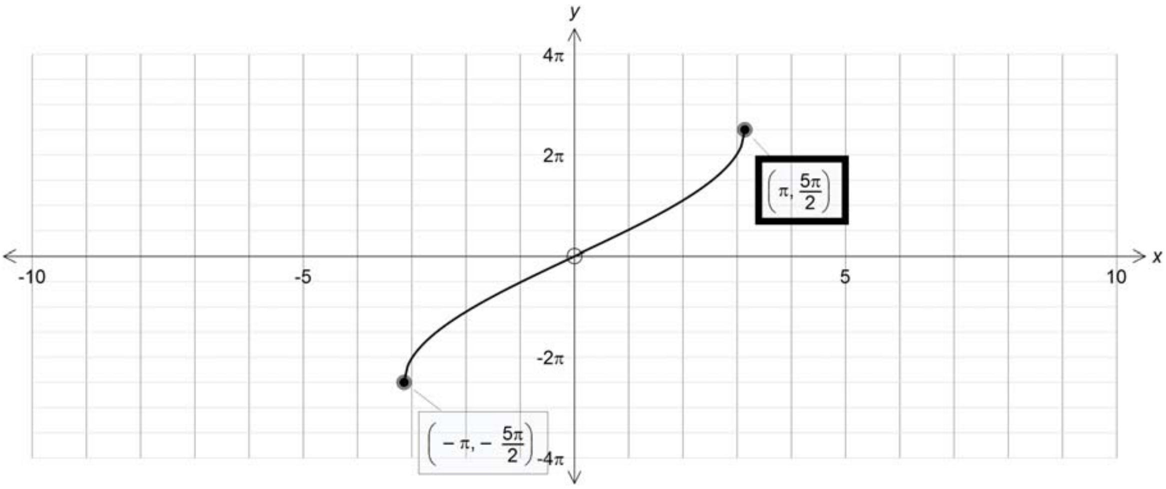
$= \frac{5\pi}{6\sqrt{3}} = \frac{5\sqrt{3}\pi}{18}$ √

(e) (i) For the function $y = 5\sin^{-1}\left(\frac{x}{\pi}\right)$.

Domain $-1 \le \frac{x}{\pi} \le 1$ so $-\pi \le x \le \pi$ √

Range $-\frac{5\pi}{2} \le y \le \frac{5\pi}{2}$ √ **2**

(ii) . **2**



13(f) For what values of x does $y = \frac{xe^x}{2}$ have an inverse function. Use differentiation to explain your answer.

$$y = \frac{xe^x}{2}$$

$$\frac{dy}{dx} = \frac{xe^x}{2} + \frac{e^x}{2} \qquad \textit{product rule}$$

$$= \frac{e^x}{2}(x + 1) \qquad \checkmark$$

$$\therefore\ x \geq -1 \quad \textit{or} \quad x \leq -1$$

$$\textit{inverse exists when } \frac{dy}{dx} < 0 \ \textit{or} \ \frac{dy}{dx} > 0 \qquad \checkmark$$

QUESTION 14 *(15 marks)* ***START A NEW BOOKLET*** **Marks**

(a)

$5a + b + 125 = 0$ ①

$-5a + b - 65 = 0$ ②

$2b + 60 = 0$ ① + ②

$2b = -60$

$b = -30$

$5a - 30 + 125 = 0$

$5a = -95$

$a = -19$ √ √ √

(b) The polynomial equation $x^3 - 5x^2 + 7x + 5 = 0$ has 3 roots, $\alpha,\ \beta,\ \gamma$

(i) $\alpha + \beta + \gamma = -\dfrac{b}{a} = 5$ √

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a} = 7$ √

(iii) $(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\beta\gamma + 2\gamma\alpha$ √ √

$\therefore \alpha^2 + \beta^2 + \gamma^2 = 5^2 - 2 \times 7 = 11$

(c) Given that $x^3 - 3x^2 + 2x - 4 = \left(x^2 + 2\right) \times Q(x) + R(x)$. **3**

Find $Q(x)$ and $R(x)$.

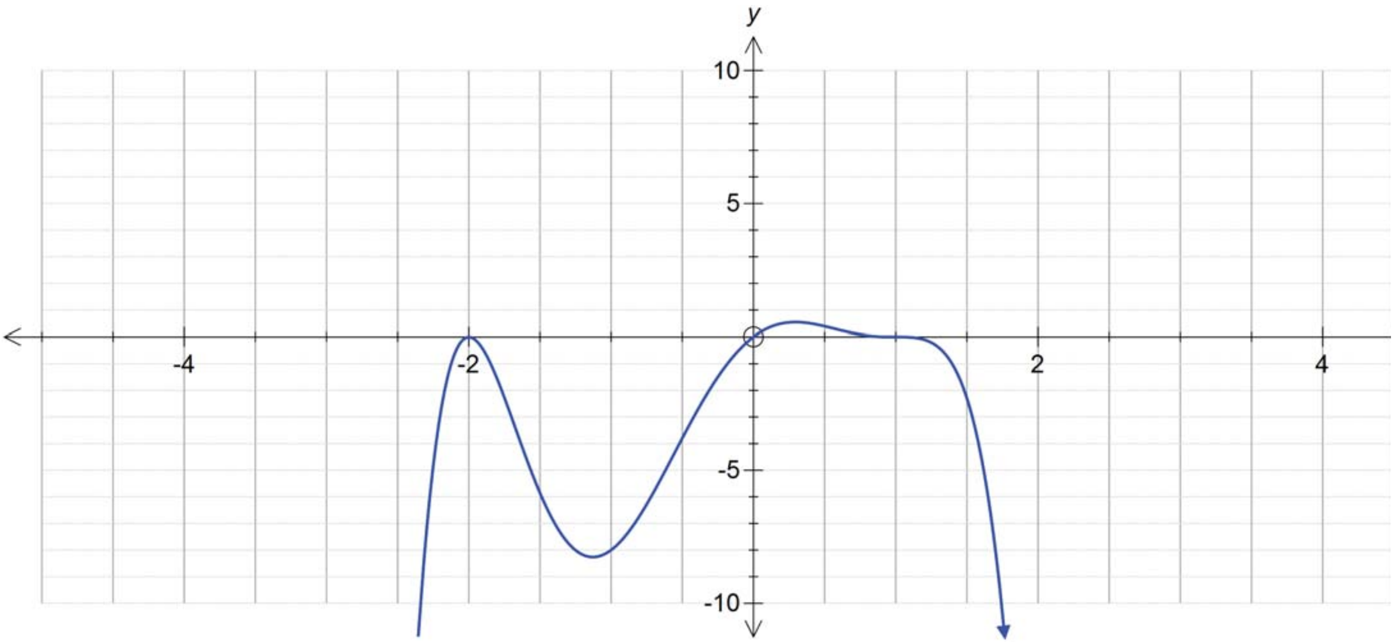
$x^3 - 3x^2 + 2x - 4$ is divided by $x^2 + 2$

$$\begin{array}{r l}
 & x - 3 \\
x^2 + 2 \,\big) & x^3 - 3x^2 + 2x - 4 \\
 & \underline{x^3 \qquad\quad + 2x \quad} \\
 & \quad - 3x^2 \qquad - 4 \\
 & \quad \underline{- 3x^2 \qquad - 6} \\
 & \qquad\qquad\quad 2
\end{array}$$

$Q(x) = x - 3$ and $R(x) = 2$ √ √ √

(d) Sketch the following polynomial. Clearly show all intercepts.

$P(x) = x(x+2)^2(1-x)^3$. √ √



√ √

$14(e)$ *let* *roots be* $\alpha, \frac{4}{\alpha}, \beta$ *ACCEPT OTHER METHODS.*

$\therefore$ *product* $\alpha \times \frac{4}{\alpha} \times \beta = -\frac{12}{3}$

$\beta = -1$

$\therefore$ *sum* $\alpha + \frac{4}{\alpha} + \beta = \frac{17}{3}$

$\alpha + \frac{4}{\alpha} - 1 = \frac{17}{3}$

$3\alpha^2 - 20\alpha + 12 = 0$

$(3\alpha - 2)(\alpha - 6) = 0$

roots are $-1, \frac{2}{3}$ *and* 6. √√√

END OF THE PAPER